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Effects of nonlocal elasticity and Knudsen number on fluid–structure interaction in carbon nanotube conveying fluid

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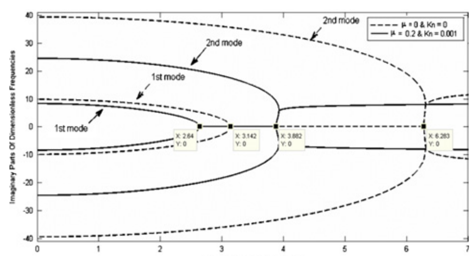
HIGHLIGHT

- ► Simultaneous small-size effect of both fluid and structure in FSI are considered.
- ► *Kn* has more remarkable effect than nonlocal parameter on gas nano-flow.
- ► Nonlocal parameter decreases second critical velocity more than the first one.
- ► In liquid flow nonlocal parameter causes more reduction in critical velocity than *Kn*.

GRAPHICAL ABSTRACT

Increase in *Kn* and nonlocal parameter, μ, advances flow instabilities in CNTs conveying fluid drastically for liquid nano-flow, as opposed to absence of those effects.



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ABSTRACT

In this paper, we investigate the effect of nano-size of both fluid flow and elastic structure simultaneously on the vibrational behavior of a pinned–pinned and a clamped–clamped nanotube conveying fluid, using both Knudsen number (*Kn*) and nonlocal continuum theory. Euler–Bernoulli plug flow (EBPF) theory is used for modeling fluid–structure interaction (FSI). It is observed that nonlocal parameter has more effect than *Kn* on the reduction of critical velocities of a liquid nano-flow. This effect has considerable impact on the reduction of critical velocities for a clamped–clamped beam in comparison with a pinned–pinned one. We concluded that the dimensionless nonlocal parameter, had more impressive effect on the dimensionless critical flow velocity of the second mode divergence and coupled mode flutter instabilities. However, in a gas nano-flow, the situation is totally different and *Kn* causes more reduction in critical velocities. Furthermore, it is emphasized that ignoring nano-size effects on liquid and gas nano-flow might cause non-conservative design of nano-devices.

1. Introduction

Carbon nanotubes (CNTs) are effectively long and thin cylinders of graphite. Due to perfect hollow cylindrical geometry and superior mechanical strength, CNTs have potential usage as cancer therapy devices or nano-vessels for conveying and storing fluids and drug delivery in bio-nanotechnology [1]. In this regard, a remarkable number of studies have been accomplished to disclose the vibrational behavior of such nano-structures. For instance, Yoon et al. [2] studied the influence of internal flow on free vibration and flow-induced structural instability of CNT. They showed that the internal moving fluid could substantially affect vibrational frequencies especially for suspended, longer and larger-innermost radius CNTs at higher flow velocities. Wang et al. [3] investigated buckling instability of double-walled CNT conveying fluid by using a multi-elastic beam model and showed that the effect of the van der Waals force, slenderness ratio and spring constant of surrounding elastic medium on the critical flow velocity were significant. Zhen et al. [4] showed that the resonant

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frequencies and critical flow velocities were significantly dependent on the properties of the surrounding medium, the boundary conditions and the temperature change. Mahdavi et al. [5] studied nonlinear vibration of an embedded double-walled carbon nanotube (DWCNT) aroused by nonlinear van der Waals (vdWs) interaction forces from both surrounding medium and adjacent tubes. They found that the nonlinear vdW forces from the surrounding medium resulted in non-coaxial vibration of the embedded DWCNT.

In fluid–structure interaction (FSI) problems, specifically in slender structures conveying axial flow, divergence (static buckling) and flutter (dynamic buckling) stabilities establish the core of discussion. For a long time, Flutter, as a post-divergence dynamic phenomenon occurring for conservative systems, i.e., for positively supported 1/D systems such as doubly hinged, doubly clamped, and hinged-clamped boundary conditions, has been a room for debate and ambiguity. In a review article by Wang and Ni [6], in 1977, Done and Simpson [7] opened up a controversial question about the capability of occurrence of post-divergence (flutter) in conservative systems such as beams supported at both end. In the same review [6], in 1978, Holmes [8] answered this question as the title of his paper, “Pipes supported at both ends cannot flutter”. His conclusion was based on nonlinear dynamic theory. From these studies, it may be concluded that without conducting a nonlinear analysis for investigating post-divergence (flutter) in FSI problems, it may be not sufficient to draw any conclusion about post-divergence phenomena solely based on a linear analysis. Since this issue is still questionable and under debate; we may disregard such a nonlinear analysis.

In this research, however we are considering small-size effects for both slip boundary conditions on fluid flow and solid structure on the vibrational behavior of CNTs. In the earlier studies, researchers only considered nano-scale effect on either slip boundary condition on fluid flow or structural system; however, in this paper, we investigate both effects simultaneously. In many recent studies, various size-dependent continuum theories have been developed for vibration and instability analysis of CNTs conveying fluid. Ke and Wang [9] investigated vibration and instability of fluid-conveying double-walled carbon nanotubes based on modified couple stress theory. They showed that the imaginary component of the frequency and the critical flow velocity of the CNTs increased with an increase in length scale parameter. Wang [10] utilized nonlocal elasticity theory integrated with surface elasticity theory to model the fluid conveying nanotubes with both inner and outer surface layers. He revealed that surface effect was substantial, especially for smaller tube thicknesses or larger aspect ratios. The fundamental frequency predicted by his new model was generally higher than that predicted by the Euler–Bernoulli beam model without surface effects. Wang [11] developed a theoretical analysis of wave propagation of fluid-conveying single-walled carbon nanotubes based on strain gradient elasticity theory. He emphasized that two small-scale parameters related to the inertia and strain gradients significantly affect phase velocity at higher wave numbers. In addition to the above-mentioned theories, most of researchers developed a nonlocal elastic beam model to analyze vibration and instability of CNTs conveying fluid, using theory of nonlocal elasticity. Tounsi et al. [12] made a comment on the work written by Lee and Chang [13] that had been about the vibration analysis of fluid-conveying double-walled carbon nanotubes based on nonlocal elastic theory. Tounsi et al. [12] corrected the equation of motion extracted by Lee and Chang [13] and rederived correct equations. Zhen and Fang [14] investigated thermal and nonlocal effects on the vibration and instability of single-walled CNT conveying fluid and indicated that the natural frequencies and critical flow velocities increased as temperature changes increased, and thermal effect could reduce the influence of nonlocal effect. Rafiei et al. [15] discussed about the effects of taper ratio and small-scale parameter on the vibration of non-uniform carbon nanotubes. They revealed that non-dimensional frequencies obtained from nonlocal theory are less than those obtained from a local theory. In addition, they showed that by increasing the taper ratio, the critical flow velocity decreased. Recently, there have been new trends in formulating nonlocal elasticity theory in engineering communities. Based on [16,17], the earlier trend has been called “partial” nonlocal elasticity, due to ignoring higher-order boundary conditions derived from a variationally consistent formulation, while using nonlocal constitutive law, as a part of equilibrium (static/dynamic), and kinematic (geometric or compatibility) relations. By the same authors, recent trend of nonlocal formulation has been named “exact” nonlocal elasticity, because higher-order terms are derived for both differential equations and boundary conditions of nonlocal boundary-value problem. Wang [18] developed the higher-order governing equation and the boundary conditions based on exact nonlocal stress model to examine the vibration properties and stability of nanotubes conveying fluid. This modified nonlocal beam model for nanotubes conveying fluid readily predicted that the natural frequencies and critical flow velocities are significantly different from those given by the partial nonlocal beam model. Actually, the trends of changes in stiffness, natural frequencies and buckling loads were observed to be completely in opposite for the two theories.

Rashidi et al. [19] presented an innovative model for a single-mode coupled vibrations of nanotubes conveying fluid by considering the small-size effects on the flow field. They formulated the small-size effects on slip boundary conditions of nano-flow through Knudsen number (Kn). They reported, for passage of gas through nano-pipe with nonzero Kn, the critical flow velocities decreased considerably as opposed to those for zero Kn and also nonzero Kn has no appreciable effect on the general behavior of CNT conveying liquid. Mirramezani and Mirdamadi [20] revealed that for a clamped-pinned conveying gas fluid, they could see the coupled-mode flutter and mode combination for a Kn higher than zero, while they could not observe this phenomenon for a Kn equal to zero and other conditions fixed.

A major objective of this study is to propose a model, for the coupled vibrations of carbon nanotubes conveying fluid, taking into account the small-size effects of both flow field and solid structure in CNTs using Knudsen number and nonlocal continuum theory. We have studied the influences of both small-size effects on the critical velocities, velocities at which divergence and flutter instabilities may occur. It could be seen that these effects had significant influences on the dimensionless critical flow velocities for both pinned-pinned and clamped–clamped boundary conditions especially when the fluid flow is a gas. For numerical solution, we have discretized pined-pined and clamped–clamped beam by choosing two generalized coordinates in order to show flutter instability in addition to divergence instability in the first and second modes of nano-beam vibrations.

The remainder of this study is organized as follows: In Section 2, we re-formulate the fluid–structure interaction (FSI) governing equations by considering small-size effects of both flow field and elastic structure. In Section 3, we implement the Galerkin weighted-residual solution technique and solve the partial differential equations of nanotube vibrations. In Section 4, we discuss about stability analysis and present the results. Finally, in Section 5, we express our conclusions.

2. Nonlocal and Kn-dependent fluid–structure interaction (FSI) equation

In this section, we devise a nonlocal FSI formulation depending on Kn. The dependency is applied by the definition of a Kn-dependent flow velocity. The conventional governing equation of motion for free vibration of a tube conveying fluid can be expressed as [21]:

$$\frac{\partial Q}{\partial x} = m_c \frac{\partial^2 W}{\partial t^2} + F_w - p, \tag{1}$$

where x is the longitudinal coordinate of tube elastic axis; m_c, the CNT mass per unit length; W is the flexural displacement of the CNT wall; t, time; p, the distributed transverse force along axis x and Q is shear force resultant on the wall cross section, that is equal to the first derivative of the resultant bending moment with respect to x $(Q = \partial M/\partial x)$.

F_w in Eq. (1), is the force per unit length induced by a plug flow. If we neglect the effects of gravity, internal damping, externally imposed tension, pressurization and ignore the effect of viscosity of flowing fluid on the vibration and instability of nanotubes according to Wang and Ni [22], F_w is given by [21]:

$$F_w = m_f \left(2V \frac{\partial^2 W}{\partial x \partial t} + V^2 \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial t^2} \right), \tag{2}$$

where m_f is defined as the fluid mass per unit length, and V is the average axial flow velocity through the pipe.

In nonlocal elasticity, the relationship between the bending moment resultant and the flexural displacement of the Euler–Bernoulli beam theory, take the following special form [23]:

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^2 W}{\partial x^2}, \tag{3}$$

where e_0 is a material constant; a, internal characteristic length; E, the beam Young's modulus and I is the moment of inertia of cross-sectional area. By substituting the first derivative of Eq. (3) with respect to x and Eq. (2) into Eq. (1), the equation of motion in the absence of distributed transverse force along axis x, can be expressed as:

$$EI \frac{\partial^4 W}{\partial x^4} + m_c \frac{\partial^2 W}{\partial t^2} + m_f \left(2V \frac{\partial^2 W}{\partial x \partial t} + V^2 \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial t^2} \right) - (e_0 a)^2 \left((m_f + m_c) \frac{\partial^4 W}{\partial x^2 \partial t^2} + 2 m_f V \frac{\partial^4 W}{\partial x^3 \partial t} + m_f V^2 \frac{\partial^4 W}{\partial x^4} \right) = 0 \tag{4}$$

The governing equations for the conventional FSI problems are basically derived by the assumption of no-slip boundary conditions. With due attention to the Kn effect on nano-size flow, this condition is no longer valid. According to Rashidi et al. [19] we can represent an average velocity correction factor (VCF).

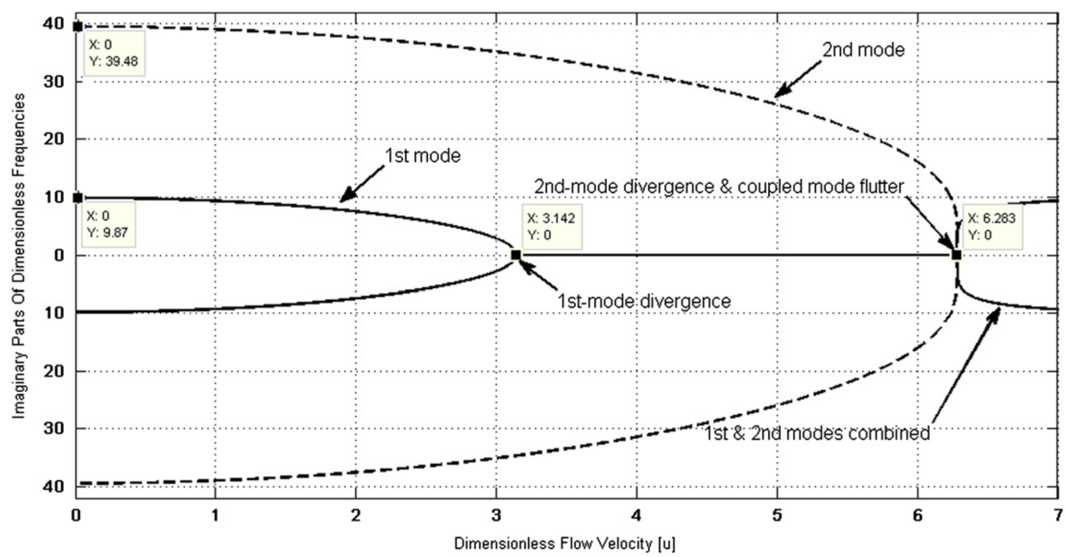

Fig. 1. Imaginary parts of dimensionless frequencies versus dimensionless velocities for a pinned–pinned CNT conveying liquid herein acetone (Kn=0 and μ=0).

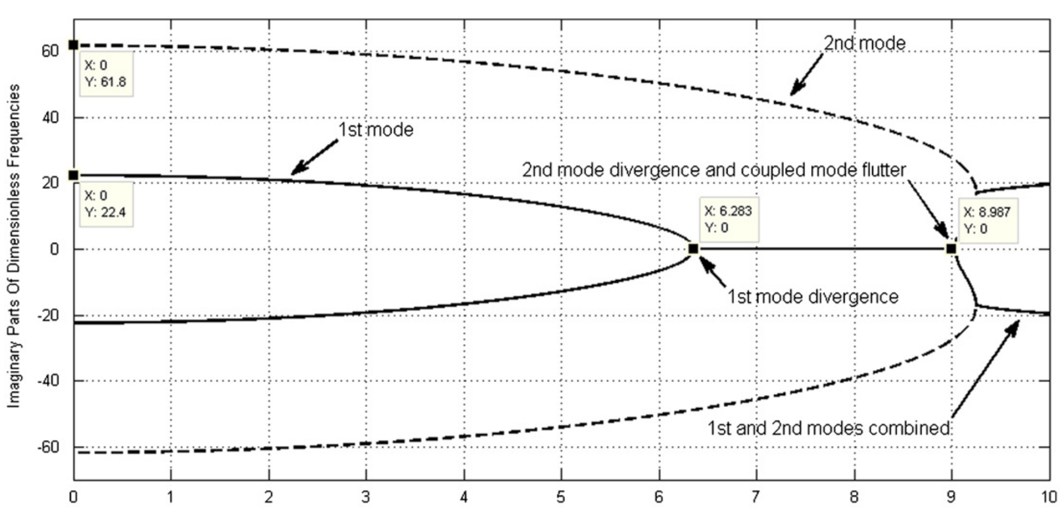

Fig. 2. Imaginary parts of dimensionless frequencies versus dimensionless velocities for a clamped–clamped CNT conveying liquid herein acetone (Kn=0 and μ=0).

The term $(1+aKn)$ appears because of considering the viscosity of the fluid flow. In our study, we neglect the effect of viscosity, so the average velocity correction factor is as follows:

$$VCF \triangleq \frac{V_{avg,slip}}{V_{avg,(no-slip)}} = \left(4\left(\frac{2-\sigma_v}{\sigma_v}\right)\left(\frac{Kn}{1+Kn}\right)+1\right) \tag{5}$$

where σ_v is tangential momentum accommodation coefficient and is considered to be 0.7 for most practical purposes [24], and Kn is the Knudsen number. From now on, by considering the average velocity correction factor, we can use Eq. (4) and it is just adequate to replace the $V_{avg,slip}$ by $(VCF) \times V_{avg,(no-slip)}$. If we follow this procedure, we reach the final dimensionless equation as:

$$\frac{\partial^4\eta}{\partial\zeta^4} + (VCF)^2 \times (u_{avg,(no-slip)})^2 \frac{\partial^2\eta}{\partial\zeta^2} + 2(VCF)\beta^{1/2} \times u_{avg,(no-slip)} \frac{\partial^2\eta}{\partial\zeta\partial\tau} + \frac{\partial^2\eta}{\partial\tau^2} - \mu^2\left((VCF)^2(u_{avg,(no-slip)})^2 \frac{\partial^4\eta}{\partial\zeta^4} + \frac{\partial^4\eta}{\partial\tau^2\partial\zeta^2} + 2(VCF)\beta^{1/2} \times u_{avg,(no-slip)} \frac{\partial^4\eta}{\partial\zeta^3\partial\tau}\right) = 0 \tag{6}$$

In our case, the following dimensionless parameters have been arisen:

$$(a): \tau = \left(\frac{EI}{m_c+m_f}\right)^{\frac{1}{2}} \frac{t}{L^2}, \quad (b): \zeta = \frac{x}{L}, \quad (c): \eta = \frac{W}{L},$$
$$(d): \beta = \frac{m_f}{m_c+m_f}, \quad (e): \mu = \frac{e_0 a}{L} \tag{7}$$

$$(a): u_{avg,slip} = (m_f/EI)^{1/2} L \times V_{avg,slip},$$
$$(b): u_{avg,(no-slip)} = (m_f/EI)^{1/2} L \times V_{avg,(no-slip)} \tag{8}$$

where η, ζ, β, u, τ, and μ are dimensionless values of lateral deflection, axial coordinate, mass parameter, axial flow velocity, time parameter, and nonlocal parameter, respectively. To the authors' knowledge, the above equation is an analytical approach to take into account for the small-scale effects both on fluid flow side and on elastic structure, using Knudsen number and nonlocal continuum theory, simultaneously.

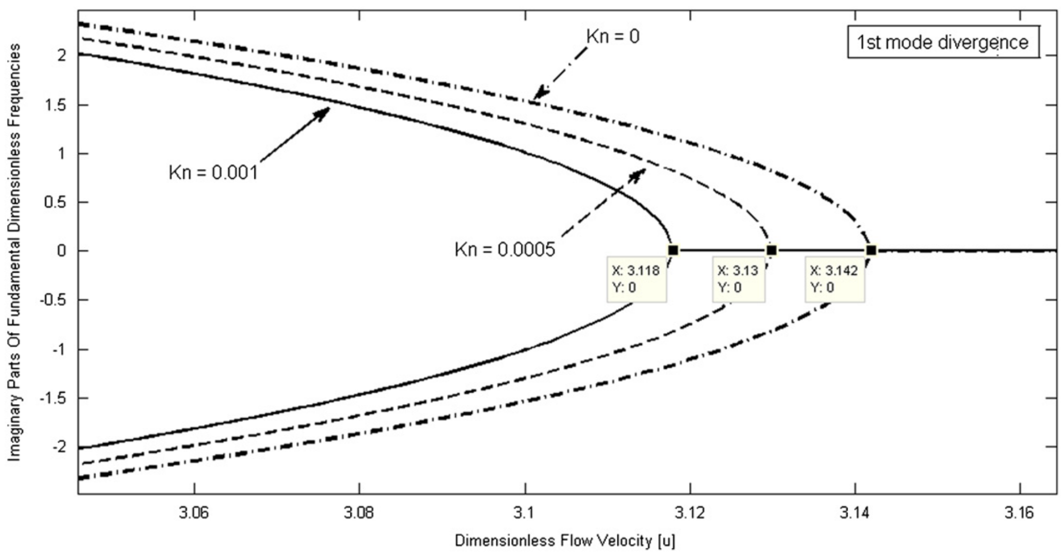

Fig. 3. Imaginary parts of first dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying liquid herein acetone (zoom in).

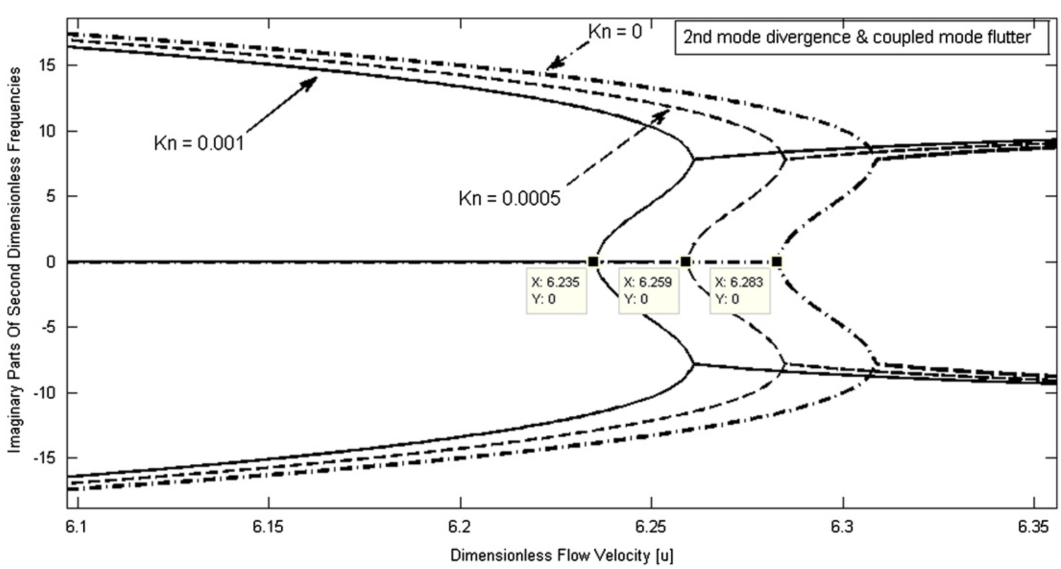

Fig. 4. Imaginary parts of second dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying liquid herein acetone (zoom in).

3. Approximation solution technique

In order to solve the above FSI equation and to calculate the CNT complex-valued eigen-frequencies, and extract the divergence and flutter instability conditions, we use Galerkin approximate solution method. This is approximate, not only in a strict numerical sense, but also because of the finite number of terms utilized in the series solution expansion. Let

$$\eta(\zeta,\tau) \cong \sum_{n=1}^{N} \Phi_n(\zeta) q_n(\tau). \tag{9}$$

where $q_n(\tau)$ are the generalized coordinates of the discretized system and $\Phi_n(\zeta)$ are the dimensionless eigen-functions of a beam with the same boundary conditions as the nano-pipe under consideration. It is presumed that the series may be truncated at a suitably high value of n. It should be noted that in this approximate method we need to choose dimensionless eigen-functions to satisfy the essential and natural boundary conditions of a nonlocal pinned–pinned ($\eta=0$ and $\partial^2\eta/\partial\zeta^2$ at $\zeta=0$ and $\zeta=1$) and a nonlocal clamped–clamped ($\eta=0$ and $\partial\eta/\partial\zeta=0$ at $\zeta=0$ and $\zeta=1$) beam [23].

We may assume the generalized coordinates $q_n(\tau), n=1,2,\ldots,N$ vary as simple harmonic motion (SHMs) for a free vibration response:

$$q_n(\tau) = Q_n \exp(s_n \tau) \tag{10}$$

where Q_n are dimensionless constant amplitudes of nth generalized coordinate of CNT free vibration and s_n are complex-valued eigen-frequencies, whose real parts show the decaying rate in the nth mode (modal equivalent viscous damping) and the imaginary parts show the nth modal damped natural frequency of the beam nano-structure. Furthermore, according to Eq. (7) the

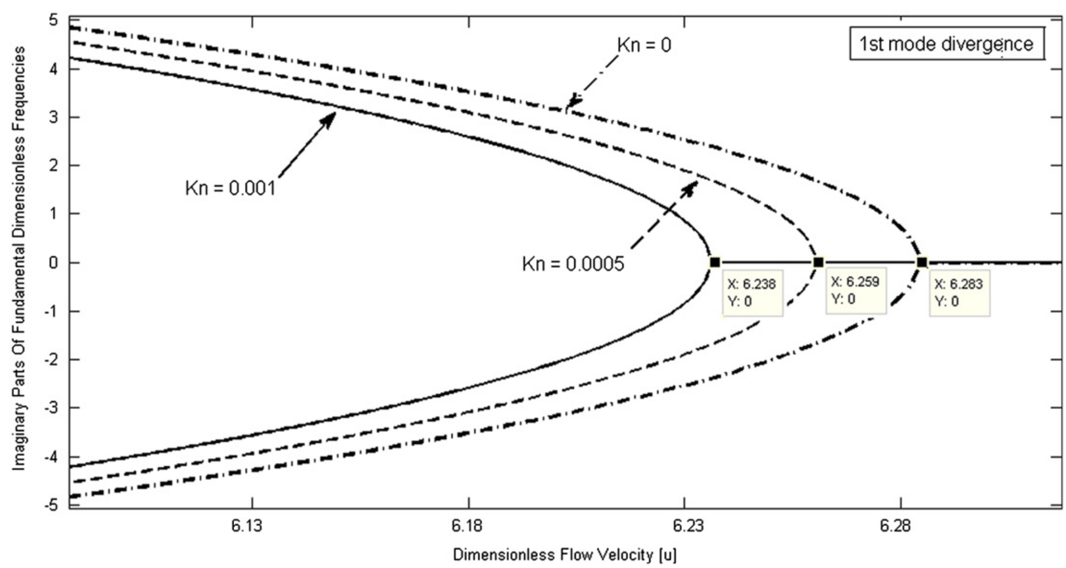

Fig. 5. Imaginary parts of first dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying liquid herein acetone (zoom in).

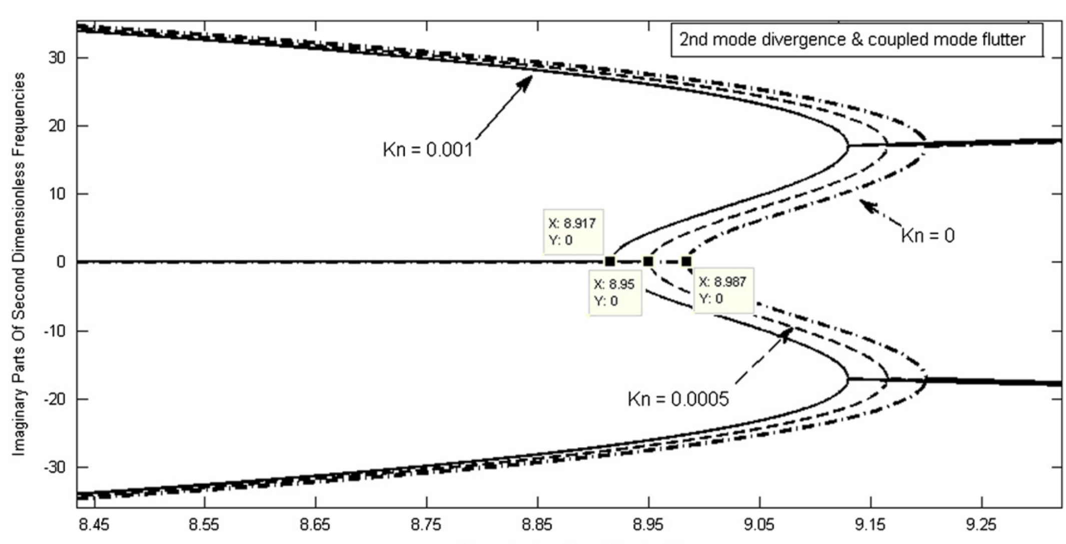

Fig. 6. Imaginary parts of second dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying liquid herein acetone (zoom in).

dimensionless eigen-frequencies Ω_n are related to the eigen-frequencies (in terms of rad/s) as follows:

$$\Omega_n = \left(\frac{m_f + m_c}{EI}\right)^{\frac{1}{2}} L^2 s_n \tag{11}$$

4. Results and discussions

In this section, we utilize the approximate solutions of Galerkin method for simulating numerically the behavior of fluid passing through a nano-pipe. All of the figures and our results are extracted from dimensionless parameters. i.e., dimensionless velocities and dimensionless eigen-frequencies. We study the effects of *Kn* number and dimensionless nonlocal parameters on the critical flow velocities, for a liquid and a gas flow through CNTs. We consider the material property of nanotube as, $\rho_{CNT}=2.3\ \mathrm{g\ cm^{-3}}$. The fluid mass density is $\rho_l=0.79\ \mathrm{g\ cm^{-3}}$ for acetone, and $\rho_g=0.001169\ \mathrm{g\ cm^{-3}}$ for air.

4.1. Validation of numerical solution

In this subsection, we compare our numerical results with those of Paidoussis [25]. Fig. 1 and Fig. 2 show how the imaginary parts of the first two dimensionless eigen-frequencies of a pipe will change for various values of dimensionless average flow velocity of acetone, through a pinned–pinned and a clamped–clamped pipe. In this case, the average flow velocity is assumed based on classical Navier–Stokes' continuum mechanics; i.e., *Kn* is assumed to be zero. We observe from these two figures that as the mean flow velocity increases from zero to a critical value, the resonant frequencies approach zero. For critical flow velocities, the resonant frequencies become zero; consequently, the pipe stiffness disappears, and divergence or column buckling occurs. As we observe from Fig. 1, in the case of pinned–pinned beam, for a zero fluid velocity, the dimensionless resonant frequencies of the pipe are π^2 and $(2\pi)^2$. These values for zero flow velocity are exactly the natural (undamped) frequencies of the pipe in free vibrations. From this figure, we could see that the dimensionless critical flow velocity for the first mode divergence would be is π and that of the second mode

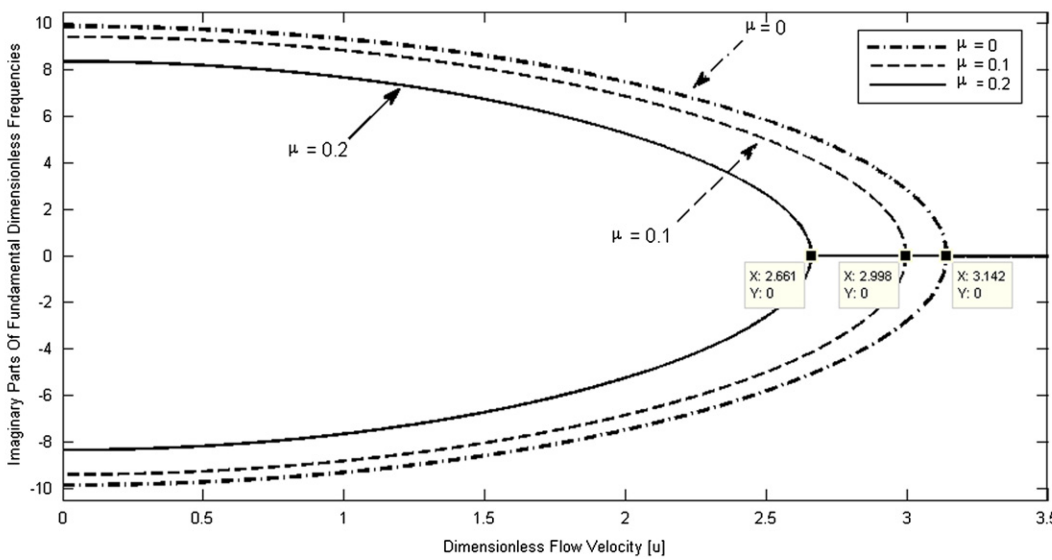

Fig. 7. Imaginary parts of first dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying liquid herein acetone ($Kn=0$).

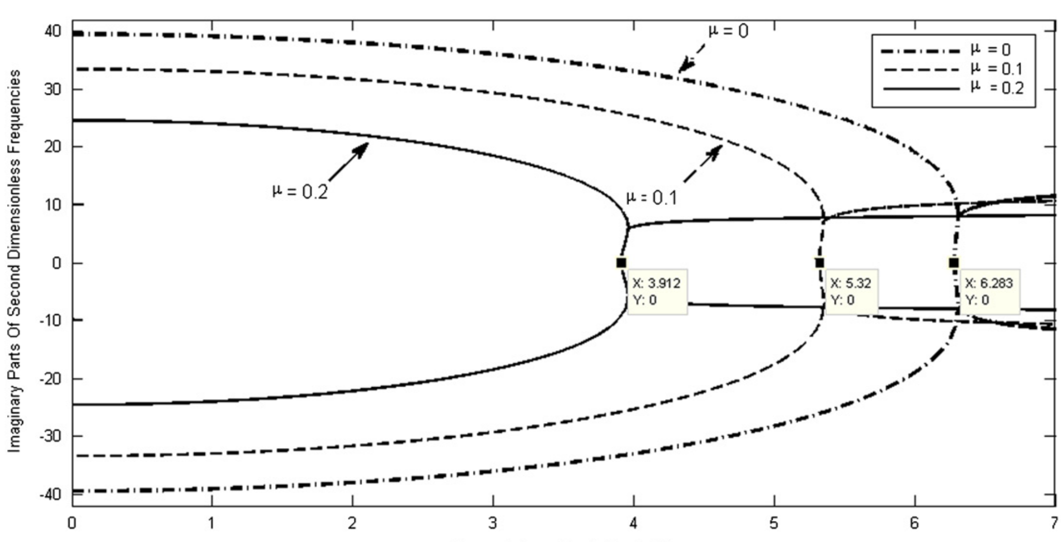

Fig. 8. Imaginary parts of second dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying liquid herein acetone ($Kn=0$).

divergence, 2π, as we would expect from the observations of Paidoussis [25].

Fig. 2, shows the same results for a clamped–clamped beam. In this figure, we could see that the dimensionless critical flow velocity for the first and second mode divergences would be 2π and 8.987, respectively. According to Paidoussis [25] for a clamped–clamped beam, the critical average flow velocities are 2π and 8.99. If we would compare these values of Paidoussis [25] by those of Fig. 1 and Fig. 2, we could find the degree of accuracy of our studies on which we could rely.

4.2. Effect of Kn on liquid nano-flow

In this subsection, we investigate the effect of a nonzero *Kn* on the dynamic response of a nano-pipe conveying a liquid, herein, acetone. According to Rashidi et al. [19] in a liquid nano-flow, *Kn* values would range from 0 to 0.001. Fig. 3 and Fig. 4, illustrate the first and second mode divergence instability of pinned–pinned nanotube conveying acetone, for three values of *Kn*, respectively. Due to little differences, we have drawn the magnified parts around the first and second mode divergence instabilities. By comparing the values of critical flow velocities for *Kn*s equal to 0.0005 and 0.001, with zero *Kn* (where we have a condition of continuum flow regime), we notice that, *Kn* has no appreciable effect on the general response of the coupled fluid-structure dynamics except that a higher *Kn* may cause an unnoticed positive shift in the divergence happening. We see that the divergence and coupled mode flutter phenomena may happen in a lower critical flow velocity, for a higher *Kn*. However, the differences are not so critical for a liquid nano-flow with a *Kn* at most equal to 0.001.

Fig. 5 and Fig. 6, illustrate the same phenomena and results for a clamped–clamped beam by considering the same range for *Kn*. According to these four figures, we observe that the maximum value of reduction is only approximately 0.8% of reduction for both different boundary conditions.

4.3. Effect of nonlocal parameter on liquid nano-flow

In this subsection, we consider the effect of nonlocal parameter on the vibrational behavior of carbon nanotubes conveying

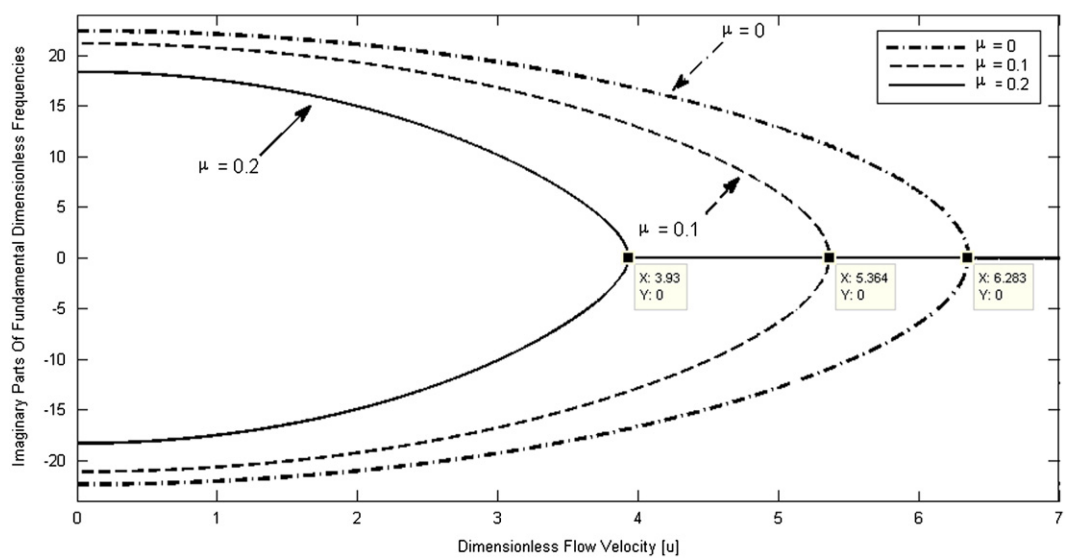

Fig. 9. Imaginary parts of first dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying liquid herein acetone ($Kn=0$).

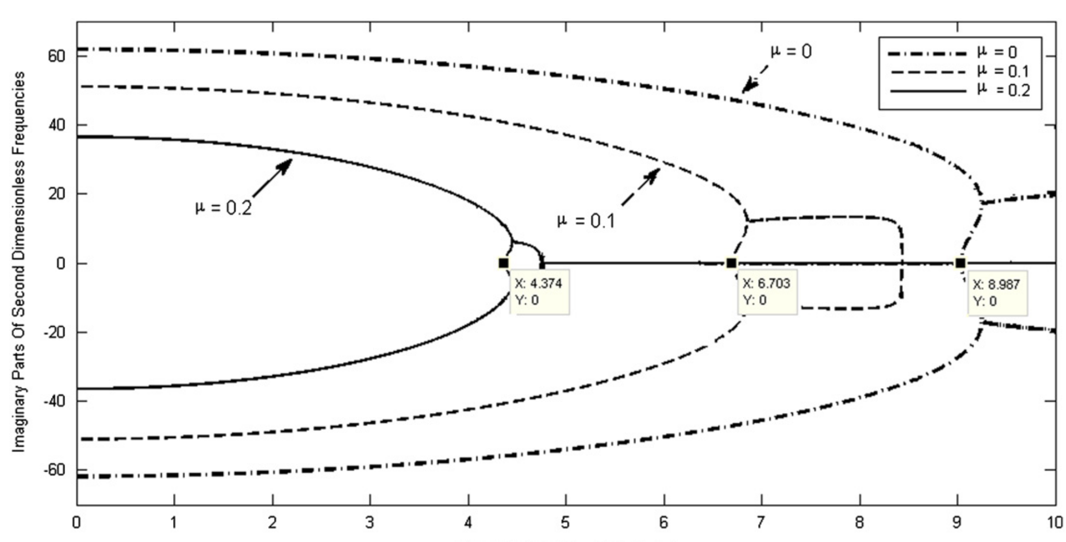

Fig. 10. Imaginary parts of second dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying liquid herein acetone ($Kn=0$).

fluid in a continuum flow regime ($Kn=0$). Before going to the details of figures, we ought to discuss about the range of the dimensionless nonlocal parameter we have chosen in our studies. According to Tounsi et al. [26] large range of values for the coefficient e_0a is possible, but for determining more accurate results for nano-beams, more experimental tests are required. In our studies we use e_0a between zero and 2 nm ($0<e_0a<2$ nm) according to Wang et al. [27]. However, the dimensionless nonlocal parameter μ, according to the range of e_0a can be ignored for micro-beams. Furthermore, in our work the length of the beam is in a nano-scale and the range of μ is between 0 and 0.2 ($0<\mu<0.2$).

Fig. 7 and Fig. 8, illustrate the first and second mode divergence instability of pinned–pinned nanotube conveying acetone, for three values of μ, respectively. By comparing the values of critical flow velocities for $\mu=0.1$ and $\mu=0.2$ with zero μ (where we have a local or continuum condition), we notice that, nonlocal elasticity has more effect on the general response of the coupled fluid-structure dynamics, in comparison with the effect of Kn. Higher value of μ may cause sooner divergence and flutter instabilities to happen. We observe from these two figures that the dimensionless nonlocal parameter μ, has more remarkable effect on the dimensionless critical flow velocity of the second mode of divergence. For maximum value of $\mu(\mu=0.2)$, we have approximately 37.74 percentage reduction in the magnitude of critical flow velocity for the second mode, and this value is about 15.31% for the first mode.

Fig. 9 and Fig. 10, illustrate the same phenomena for a clamped–clamped beam. Like a pinned–pinned system, considering the nonlocal effect reduce the critical flow velocity for both the first and the second mode. This parameter has considerable effect in the reduction of critical velocity for a clamped–clamped beam in comparison with a pinned–pinned one. In the case of clamped–clamped condition, the maximum reduction values of the magnitude of critical flow velocities are approximately 37.45 and 51.33%, for the first and second mode divergence, respectively. Similar to simply supported system, this nonlocal effect is more sensitive to the second mode.

4.4. Simultaneous effects of nonlocal parameter and Kn on liquid nano-flow

In this part, we consider the effect of both dimensionless parameters, simultaneously on the critical flow velocities. Fig. 11

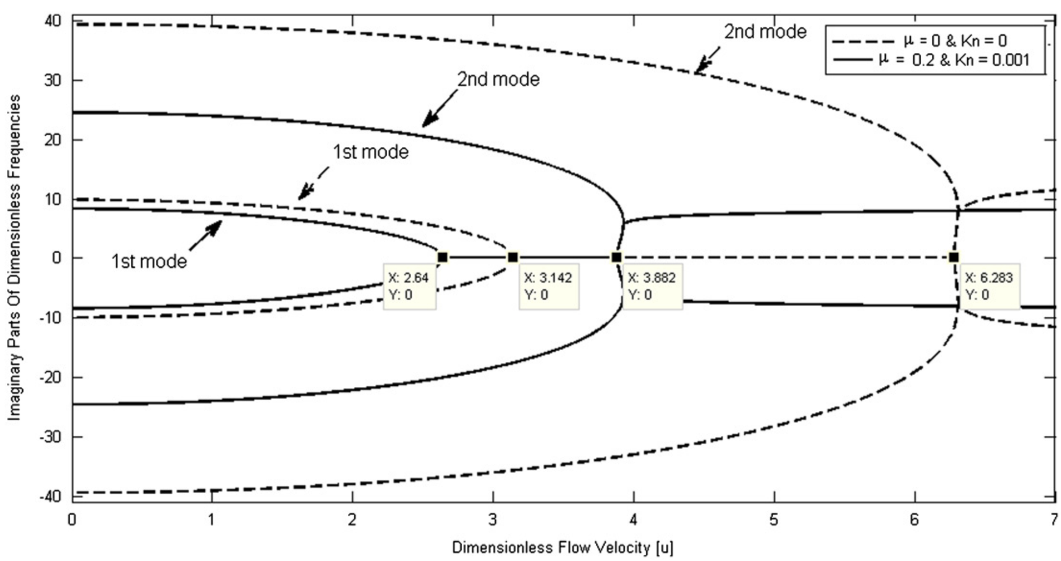

Fig. 11. Imaginary parts of dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying liquid herein acetone.

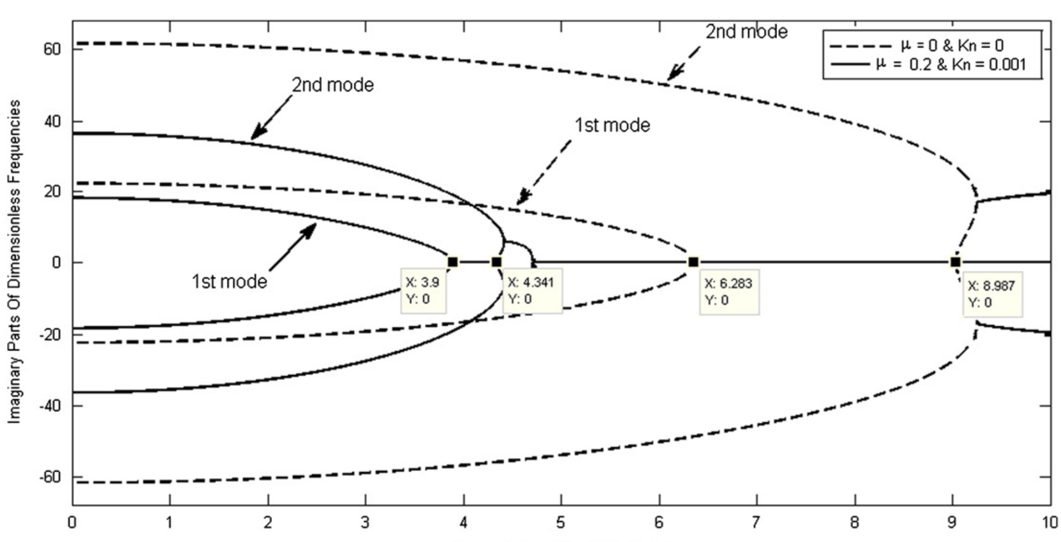

Fig. 12. Imaginary parts of dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying liquid herein acetone.

shows these effects for a pinned–pinned case. We can observe from this figure that approximately 16 and 38.21% reduction occur in the magnitudes of critical flow velocities. These values are more than the values calculated by considering each effect separately. It should be noted that when we draw the imaginary parts of dimensionless frequencies versus dimensionless flow velocities for the especial values of $\mu=0.2$ and $Kn=0.001$, the magnitude of reduction in the first and second critical mean velocities are, 0.502 and 2.401, respectively. These critical values in a condition of $\mu=0.2$ and $Kn=0$ are respectively, 0.481 and 2.371, for the first and second modes. Also, in a case in which, $\mu=0$ and $Kn=0.001$, the amount of reduction in the first mode is 0.024, and in the second mode is 0.048. With due attention to the fact that the values of reduction in the first and second modes are not equal to the addition of the values of reduction in each case, where we are considering both effects simultaneously, then, regarding only one of the effects (for example in the first mode, 0.502 QUOTE $0.481+0.024$), we may conclude that there is not a linear relationship between the effects of these dimensionless parameters. However, we may only say, in a liquid nano-flow, the dimensionless nonlocal parameter plays a more important role in comparison with *Kn*. Indeed, the nonlocal effect has more contribution than *Kn* in the reduction of critical velocities.

Fig. 12 illustrates that 37.93 and 51.7% reduction occur for a clamped–clamped nano-pipe. Like a pinned–pinned system, these values are more than the values calculated by considering each effect separately. These parameters also have remarkable effect in the reduction of critical velocities for a clamped–clamped beam in comparison with a pinned–pinned one. All of the results and discussions mentioned for a simply supported beam are also valid for these boundary conditions.

4.5. Effect of Kn on gas nano-flow

In this subsection, we consider the effect of *Kn* on vibrational response of a nano-pipe conveying a gas, herein, air. Fig. 13 and Fig. 14 compare the evolutions of imaginary parts of the vibration frequencies of a simply supported and a clamped–clamped nano-pipe against the flow velocity for two different values of *Kn*, respectively.

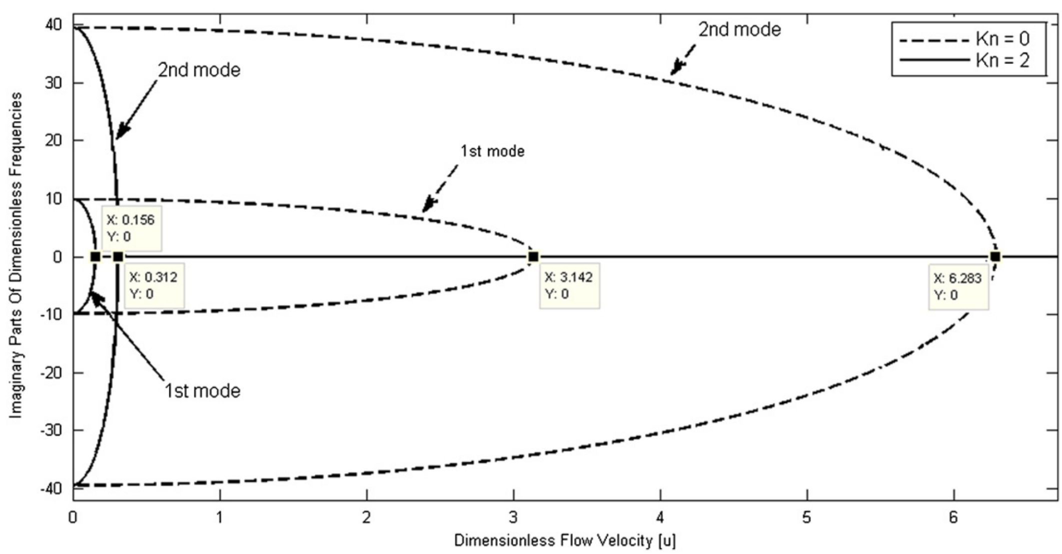

Fig. 13. Imaginary parts of dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying gas herein air ($\mu=0$).

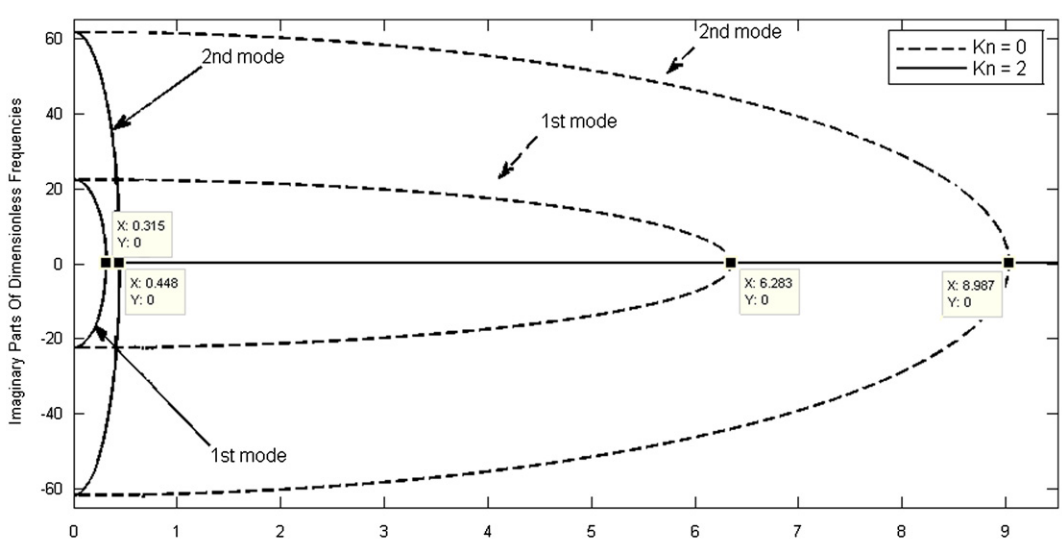

Fig. 14. Imaginary parts of dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying gas herein air ($\mu=0$).

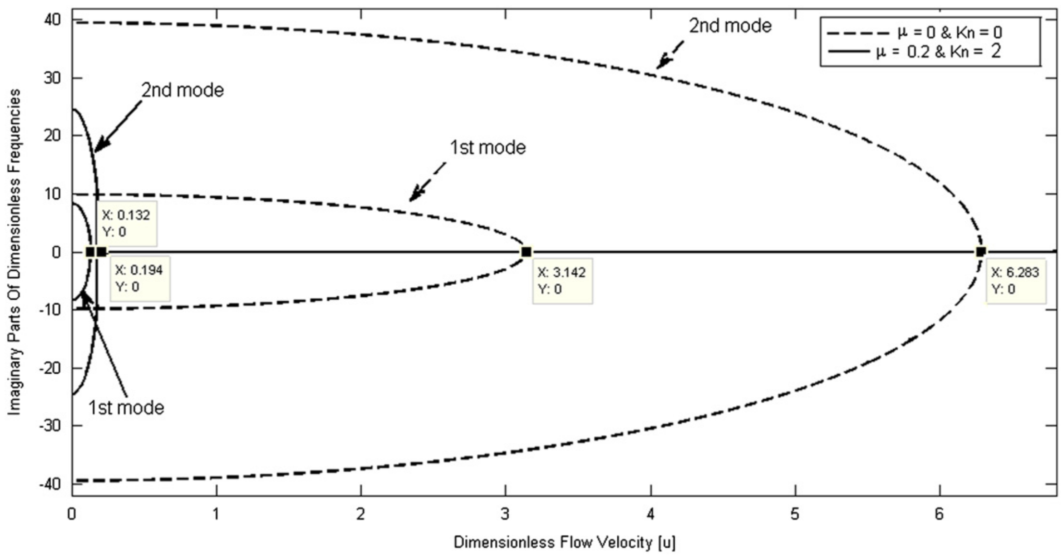

Fig. 15. Imaginary parts of dimensionless frequencies versus dimensionless flow velocities for a pinned–pinned CNT conveying gas herein air.

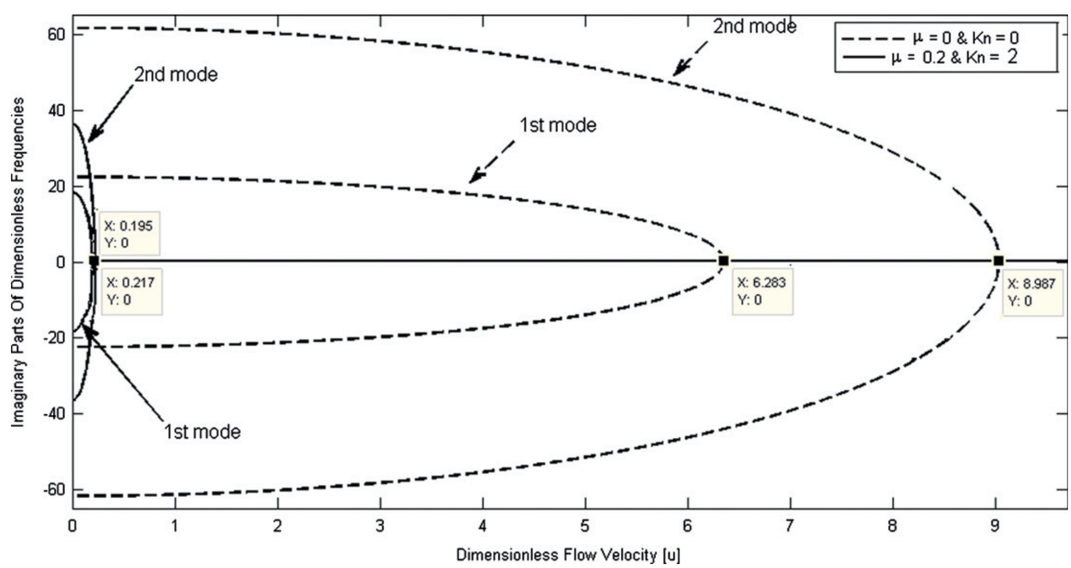

Fig. 16. Imaginary parts of dimensionless frequencies versus dimensionless flow velocities for a clamped–clamped CNT conveying gas herein air.

According to Rashidi et al. [19] in a gas nano-flow, *Kn* values can vary from 0 to 2. By comparing the values of critical flow velocities for *Kn* equal to 2 with *Kn* equal to zero, we realize that the relative maximum value of reduction is approximately 95% for two different boundary conditions. This could show that ignoring *Kn* effect on a gas nano-flow might cause erroneously a non-conservative structural design of fluidic nano-devices.

Yet, the critical gas flow velocities would reduce more drastically than those of liquid flow, by increasing *Kn* from zero to two. It should be noted, that *Kn* would have the same effects on two different boundary conditions and also the first and second mode divergence instabilities of each boundary condition which is the same as the effect of *Kn* on liquid nano-flow.

4.6. Simultaneous effects of nonlocal parameter and Kn on gas nano-flow

The range of nonlocal parameter is independent of the liquid and gas nano-flow and it shows the nano-scale effect on an elastic structure, herein a CNT. Furthermore, considering the nonlocal parameter causes the same reduction in critical velocities for a gas and liquid nano-flow. Finally, in our recent work we study the influence of both effects simultaneously for a CNT conveying a gas fluid. As we may see from Fig. 15, adjusting μ as 0.2 and *Kn* as 2, causes an extraordinary reduction of the average critical velocities for a pinned–pinned CNT, i.e., 95.8 and 96.91% reduction in the first and second modes, respectively. According to the discussion in section 4.4 and the results of four recent figures, we can easily guess that in a gas nano-flow, *Kn* has more contribution in the reduction of critical velocities than nonlocal effect where in a liquid nano-flow the situation is totally different and the nonlocal parameter has more effects. As it could be seen from Fig. 16, the amount of reduction in the first mode is 96.9, and of the second mode 97.59, for a gas nano-flow with clamped–clamped boundary condition for CNT.

5. Conclusions

In this study, we investigated small-size effects of both flow field and the elastic structure, on the dynamical behavior of a nano-channel conveying fluid using Knudsen number and nonlocal continuum theory. Effectively, we replaced the classical definition of flow velocity by a *Kn*-dependent flow velocity by using VCF parameter in our equations. We applied Galerkin method of weighted residuals to transform our BVP of fluid–structure interaction, including a PDE and the corresponding geometric and natural boundary conditions, into a system of discretized ODEs. We simulated those ODEs with two generalized coordinates for clamped–clamped and pinned–pinned boundary conditions. The effect of *Kn* on liquid nano-flow for both boundary conditions of clamped–clamped and pinned–pinned is the same and we observed that the effect of slip condition on a liquid nano-flow was not an influential parameter on the nano-flow behavior with respect to a continuum flow regime ($Kn=0$). Then, we showed that nonlocal elasticity has more effect on the vibrational response of the fluid–structure interaction, in comparison with the effect of *Kn*. This effect has considerable impact in the reduction of critical velocities for a clamped–clamped beam in comparison with a pinned–pinned one. Higher values of μ may cause sooner divergence and flutter instabilities to happen. We concluded from the figures that the dimensionless nonlocal parameter μ, had more impressive effect on the dimensionless critical flow velocity of the second mode divergence and coupled mode flutter instabilities. It seemed that nonlocal parameter had more effect for stiffener structures. We revealed in a liquid nano-flow that the dimensionless nonlocal parameter played more important role, in comparison with *Kn*, in the reduction of critical velocities. In a gas nano-flow, the contribution of *Kn* parameter in the reduction of critical velocities is more than μ. However, the effect of small-size over nano-pipe was surprisingly striking for a gas nano-flow response. Finally, it should be noted that the amount of reduction in critical velocities would not be ignored when considering the small-size effects. We observed that for the passage of a gas, like air, through the nano-pipe, the critical velocities of the first and second mode divergence decreased considerably, as opposed to those for *Kn* and μ equal to zero. This could show that ignoring small-size effects on a nano-pipe might cause erroneously, a non-conservative structural design of fluidic nano-devices, especially for a nano-flow. Yet, the critical gas flow velocities for both modes of divergence instabilities reduced more drastically than those of liquid flow, by considering small-scale effects.

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